Practice Book for the GRE® Subject Test in Mathematics

This book contains

- one full-length GRE Mathematics Test
- test-taking strategies

Become familiar with

- test structure and content
- test instructions and answering procedures

Plus: Compare your practice test results with the performance of those who took the test at a GRE administration.

ets.org/gre

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Test takers with disabilities or health-related needs who need test preparation materials in an alternate format should contact the ETS Office of Disability Services at **stassd@ets.org**. For additional information, visit **www.ets.org/gre/test-takers/subject-tests/register/disability-accommodations.html**.

Overview

The *GRE*® Mathematics Test consists of approximately 66 multiple-choice questions drawn from courses commonly offered at the undergraduate level. Testing time is 2 hours and 50 minutes; there are no separately-timed sections.

This publication provides a comprehensive overview of the GRE Mathematics Test to help you get ready for test day. It is designed to help you:

- Understand what is being tested
- Gain familiarity with the question types
- Review test-taking strategies
- Understand scoring
- Practice taking the test

To learn more about the GRE Subject Tests, and their computer-based administration beginning in September 2023, visit www.ets.org/gre/test-takers/subject-tests/about.html.

Test Content

Approximately 50 percent of the Mathematics Test questions involve calculus and its applications — subject matter that is assumed to be common to the backgrounds of almost all mathematics majors. About 25 percent of the questions in the test are in elementary algebra, linear algebra, abstract algebra, and number theory. The remaining questions deal with other areas of mathematics currently studied by undergraduates in many institutions.

The following content descriptions may assist students in preparing for the test. The percentages given are estimates; actual percentages will vary somewhat from one edition of the test to another.

I. Calculus (50%)

Material learned in the usual sequence of elementary calculus courses — differential and integral calculus of one and of several variables — including calculus-based applications and connections with coordinate geometry, trigonometry, differential equations, and other branches of mathematics

II. Algebra (25%)

- Elementary algebra: basic algebraic techniques and manipulations acquired in high school and used throughout mathematics
- Linear algebra: matrix algebra, systems of linear equations, vector spaces, linear transformations, characteristic polynomials, and eigenvalues and eigenvectors
- Abstract algebra and number theory: elementary topics from group theory, theory of rings and modules, field theory, and number theory

III. Additional Topics (25%)

- Introductory real analysis: sequences and series of numbers and functions, continuity, differentiability and integrability, and elementary topology of $\mathbb{R}$ and $\mathbb{R}^n$
- Discrete mathematics: logic, set theory, combinatorics, graph theory, and algorithms
- Other topics: general topology, geometry, complex variables, probability and statistics, and numerical analysis

The above descriptions of topics covered in the test should not be considered exhaustive; it is necessary to understand many other related concepts. Prospective test takers should be aware that questions requiring no more than a good precalculus background may be quite challenging; such questions can be among the most difficult questions on the test. In general, the questions are intended not only to test recall of information, but also to assess the understanding of fundamental concepts and the ability to apply those concepts in various situations.

Preparing for the Test

GRE Subject Test questions are designed to measure skills and knowledge gained over a long period of time. Although you might increase your scores to some extent through preparation a few weeks or months before you take the test, last minute cramming is unlikely to be of further help. The following information may be helpful.

- A general review of your college courses is probably the best preparation for the test. However, the test covers a broad range of subject matter, and no one is expected to be familiar with the content of every question.
- Become familiar with the types of questions in the GRE Mathematics Test, paying special attention to the directions. If you thoroughly understand the directions before you take the test, you will have more time during the test to focus on the questions themselves.

Test-Taking Strategies

The questions in the practice test illustrate the types of multiple-choice questions in the test.

Following are some general test-taking strategies you may want to consider.

- Read the test directions carefully, and work as rapidly as you can without being careless. For each question, choose the best answer from the available options.
- All questions are of equal value; do not waste time pondering individual questions you find extremely difficult or unfamiliar.
- You may want to work through the test quickly, first answering only the questions about which you feel confident, then going back and answering questions that require more thought, and concluding with the most difficult questions if there is time.
- Your score will be determined by the number of questions you answer correctly. Questions you answer incorrectly or for which you mark no answer or more than one answer are counted as incorrect. Nothing is subtracted from a score if you answer a question incorrectly. Therefore, to maximize your score it is better for you to guess at an answer than not to respond at all.

What Your Scores Mean

The number of questions you answered correctly on the entire test (total correct score) is converted to the total scaled score for score reporting. This conversion ensures that a scaled score reported for any edition of a GRE Mathematics Test is comparable to the same scaled score earned on any other edition of the test. Thus, equal scaled scores on a particular test indicate essentially equal levels of performance regardless of the test edition taken.

GRE Mathematics Test total scaled scores are reported on a 200 to 990 score scale in ten-point increments.

Total scaled scores should be compared only with other scores on the Mathematics Test. For example, a 680 on the Mathematics Test is not equivalent to a 680 on the Physics Test.

Taking the Practice Test

The practice test begins on page 6. The total time that you should allow for this practice test is 2 hours and 50 minutes.

It is best to take this practice test under timed conditions. Find a quiet place to take the test and make sure you have a minimum of 2 hours and 50 minutes available.

Before you begin the practice test, gather a few sheets of scratch paper for your notes and calculations during the test. When you are ready to begin the practice test, note the time and begin marking your answers in the test. Stop working on the test when 2 hours and 50 minutes have elapsed.

Scoring the Practice Test

The worksheet on page 34 lists the correct answers to the questions on the practice test. The "Correct Response" columns are provided for you to mark those questions for which you chose the correct answer.

Mark each question that you answered correctly. Then, add up your correct answers and enter your total number of correct answers in the space labeled "Total Correct" at the bottom of page 34. Next, use the "Total Score" conversion table on page 35 to find the corresponding total scaled score. For example, suppose you chose the correct answers to 50 questions on the test. The "Total Correct" entry in the conversion table of 50 shows that your total scaled score is 800.

Evaluating Your Performance

Now that you have scored your test, you may wish to compare your performance with the performance of others who took this test.

The data in the worksheet on page 34 are based on the performance of a sample of the test takers who took the GRE Mathematics Test in the United States.

The numbers in the column labeled "P+" on the worksheet indicate the percentages of examinees in this sample who answered each question correctly. You may use these numbers as a guide for evaluating your performance on each test question.

Interpretive data based on the scores earned by a recent cohort of test takers are available on the GRE website at www.ets.org/gre/test-takers/subject-tests/scores/understand-scores.html. The interpretive data shows, for each scaled score, the percentage of test takers who received lower scores. To compare yourself with this population, look at the percentage next to the scaled score you earned on the practice test. Note that these interpretive data are updated annually and reported on GRE score reports.

It is important to realize that the conditions under which you tested yourself will not be exactly the same as those you will encounter during your actual test administration. It is impossible to predict how different test-taking conditions will affect test performance, and this is only one factor that may account for differences between your practice test scores and your actual test scores. By comparing your performance on this practice test with the performance of other individuals who took GRE Mathematics Test, however, you will be able to determine your strengths and weaknesses and can then plan a program of study to prepare yourself for taking the GRE Mathematics Test under standard conditions.

GRADUATE RECORD EXAMINATIONS®

MATHEMATICS PRACTICE TEST

FORM GR3768

MATHEMATICS TEST

Time—170 minutes

66 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is best and then completely fill in the corresponding space on the answer sheet.

Computation and scratch work may be done in this test book.

In this test:

(1) All logarithms with an unspecified base are natural logarithms, that is, with base e.
(2) The symbols $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ denote the sets of integers, rational numbers, real numbers, and complex numbers, respectively.

---

1. Let C denote an arbitrary constant. Then $\int e^{ex}\,dx =$

(A) $e^{ex-1} + C$ (B) $e^{ex} + C$ (C) $e^{ex+1} + C$ (D) $xe^{ex} + C$ (E) $\dfrac{e^{ex+1}}{ex+1} + C$

---

2. $$\sum_{n=0}^{\infty} \frac{(3 \log 2)^n}{n!} =$$

(A) 1 (B) 2 (C) 3 (D) 6 (E) 8

---

3. Let r and A be the radius and the area, respectively, of a circle. If r increases by 40 percent, by what percent will A increase?

(A) 40% (B) 49% (C) 80% (D) 96% (E) 130%

---

4. If a differentiable function $y(x)$ satisfies the equation $x + y^4 = 10$ for $y \neq 0$, then $\frac{dy}{dx} =$

(A) $-\frac{1}{4y^3}$ (B) $-\frac{1}{y^4}$ (C) $-\frac{x}{4y^3}$ (D) $\frac{9}{4y^3}$ (E) $\frac{10 - x}{y^4}$

---

5. Let g be a differentiable function on $\mathbb{R}$, and let h be the function defined by $h(x) = \int_0^{x^2} g(t)\,dt$ for all $x \in \mathbb{R}$. Which of the following is equal to $h'(x)$ for all $x \in \mathbb{R}$?

(A) $g\left(x^2\right)$

(B) $2xg'\left(x^2\right)$

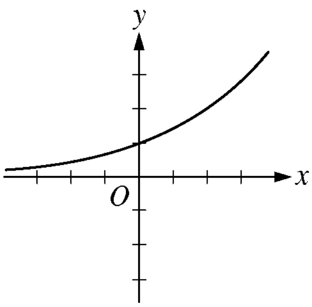
(C) $2xg\left(x^2\right)$

(D) $g'\left(x^2\right) - g'(0)$

(E) $2xg\left(x^2\right) - g(0)$

---

6. Let f be a real-valued function such that $f(f(x)) = x$ for all real numbers x in the domain of f. Which of the following could be the graph of f in the xy-plane?

(A)


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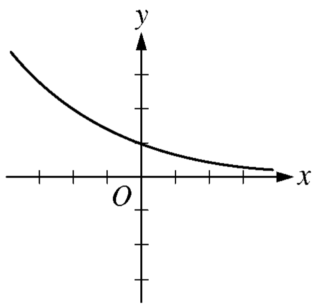

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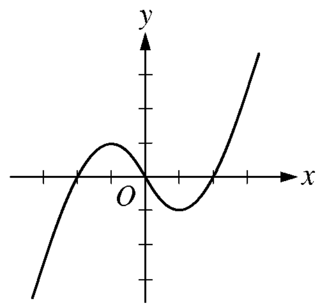

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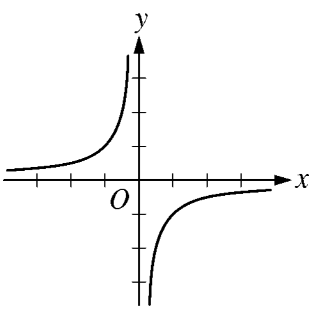
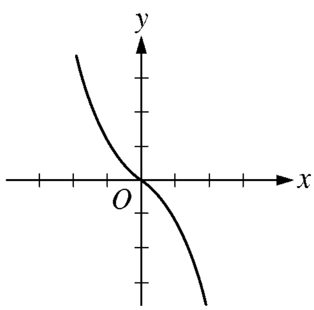


(E)


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7. If a and b are positive numbers, then $\lim_{x \to 0} \left(1 + \frac{x}{a}\right)^{b/x} =$

(A) e^{ab} (B) $e^{b/a}$ (C) $e^{a/b}$ (D) 1 (E) ∞

---

8. If the function f is defined by $f(x) = \frac{\log x}{x^2 - 1}$ for all positive numbers x except $x = 1$, and if $f(1) = a$, for what value of a is f continuous at $x = 1$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) There is no such value.

---

9. Of the 10 lightbulbs in a box, 3 are defective. If 2 lightbulbs are to be chosen from the box at random and without replacement, what is the probability that at least 1 of the 2 lightbulbs will be defective?

(A) $\frac{3}{10}$ (B) $\frac{7}{15}$ (C) $\frac{8}{15}$ (D) $\frac{19}{30}$ (E) $\frac{7}{10}$

---

10. In the power series expansion $\sum_{n=0}^{\infty} a_n x^n$ of $\frac{1}{(1-x)^2}$, where $|x| < 1$, what are the coefficients a_0, a_1, and a_2, respectively?

(A) 1, 1, and 1 (B) 1, 2, and 3 (C) 1, 2, and 6 (D) 2, 4, and 6 (E) 1, $\frac{1}{2}$, and $\frac{1}{6}$

---

11. Let $f : [-3, \infty) \to [-8, \infty)$ be defined by $f(x) = x^2 + 6x + 1$. Which of the following statements is true?

(A) f is not one-to-one (that is, not injective).

(B) f is not onto (that is, not surjective).

(C) f is one-to-one and onto, with an inverse defined on $[-8, \infty)$ by $f^{-1}(x) = 6 - \sqrt{36 + x}$.

(D) f is one-to-one and onto, with an inverse defined on $[-8, \infty)$ by $f^{-1}(x) = 6 + \sqrt{36 + x}$.

(E) f is one-to-one and onto, with an inverse defined on $[-8, \infty)$ by $f^{-1}(x) = -3 + \sqrt{8 + x}$.

---

12. Let p and q be prime numbers, where $2 < p < q$, and let M be the set of all positive integers n such that n^5 is divisible by p^3 and by $64q^{11}$. What is the least integer in M?

(A) $2pq$ (B) $4p^3q$ (C) $4pq^3$ (D) $4p^3q^3$ (E) $32pq^{10}$

---

13. Let G be a graph such that every two distinct vertices in G are connected by exactly one edge and every edge connects two distinct vertices. If G has 190 edges, how many vertices does G have?

(A) 18 (B) 19 (C) 20 (D) 94 (E) 95

---

14. Suppose f is a real-valued differentiable function defined on the interval $(0, 2)$ such that $f'(x) = x^2 - x + 1$ on $(0, 2)$. Which of the following statements must be true?

(A) If $0 < x < 2$, then $f(x) = f(2 - x)$.

(B) If $0 < x < 2$, then $f(x) = -f(2 - x)$.

(C) If $0 < x < y < 2$, then $f(x) < f(y)$.

(D) The graph of f is concave upward on $(0, 2)$.

(E) The graph of f is concave downward on $(0, 2)$.

---

15. If a and b are positive numbers, which of the following is a necessary and sufficient condition for the system of equations

$$\begin{aligned} x^2 + y^2 &= a \\ xy &= b \end{aligned}$$

to have at least one solution (x, y), where x and y are real?

(A) $a \geq 2b$ (B) $2a \leq b$ (C) $a \geq b^2$ (D) $a \leq b^2$ (E) $a \leq 2b^2$

---

16. Suppose that X and Y are sets, $f: X \to Y$ is a function, A and B are subsets of X, and C is a subset of B. Which of the following must be true?

I. $f(A \cup B) = f(A) \cup f(B)$

II. $f(A \cap B) = f(A) \cap f(B)$

III. $f(C) \subseteq f(B)$

(A) None (B) III only (C) I and II only (D) I and III only (E) I, II, and III

---

17. Which of the following statements about sequences $\{a_n\}_{n=1}^{\infty}$ of real numbers is FALSE?

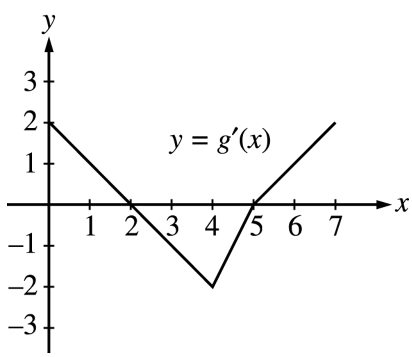
(A) Every unbounded sequence is divergent.

(B) Every bounded sequence is convergent.

(C) If $\lim_{n\to\infty} a_n = a \in \mathbb{R}$ and $f: \mathbb{R} \to \mathbb{R}$ is continuous at a, then $\lim_{n\to\infty} f(a_n) = f(a)$.

(D) If $\lim_{n\to\infty} a_n = a \in \mathbb{R}$, then for every $\varepsilon > 0$, there is a positive integer k such that $|a_m - a_n| < \varepsilon$ whenever $m, n \geq k$.

(E) For $a \in \mathbb{R}$, $\lim_{n\to\infty} a_n = a$ if and only if $\lim_{n\to\infty} |a_n - a| = 0$.



18. The graph of the derivative g' of a function g with domain $[0, 7]$ is shown above. Which of the following must be true about g ?

I. g has a local maximum value at $x = 0$ and a local minimum value at $x = 4$.

II. g has a local maximum value at $x = 2$ and a local minimum value at $x = 5$.

III. $g(2) = g(5)$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

---

19. In the xy-plane, what is the point on the curve $y = \sqrt{x+3}$ that is closest to the origin?

(A) $(-3, 0)$ (B) $\left(-\frac{3}{2}, \frac{\sqrt{6}}{2}\right)$ (C) $\left(-\frac{1}{2}, \frac{\sqrt{10}}{2}\right)$ (D) $(0, \sqrt{3})$ (E) $\left(\frac{1}{2}, \frac{3}{2}\right)$

---

20. If f and g are twice-differentiable functions of a real variable, then the second derivative of the composition $f \circ g$ is given by which of the following?

(A) $f'' \circ g''$

(B) $(f'' \circ g)(g')^2$

(C) $(f'' \circ g) + (2f' \circ g') + (f \circ g'')$

(D) $(f'' \circ g)(g')^2 + (f' \circ g)g''$

(E) $(f'' \circ g)g' + (f' \circ g')(g')^2$

---

21. If x and y are integers such that $3x + 2y \equiv 5 \pmod{13}$ and $x + 7y \equiv 1 \pmod{13}$, then $5x + 3y$ is congruent modulo 13 to

(A) 2 (B) 5 (C) 7 (D) 10 (E) 11

---

22. Consider triangle ABC with sides $AB = 6$, $AC = 8$, and $BC = 12$. Let P be a point on side AB such that $AP = 4$, and let Q be a point on side AC such that angles APQ and ACB are congruent. What is the length of line segment PQ?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 8

$$x_1 + x_4 = a$$
$$x_2 + x_4 = b$$
$$x_3 + x_4 = c$$

23. In the system of equations in the real variables x_1, x_2, x_3, and x_4 above, a, b, and c are real constants. What is the set of all $(a, b, c) \in \mathbb{R}^3$ for which the system is consistent?

(A) $\{(a, b, c) : a = b = c\}$

(B) $\{(a, b, c) : a + b + c = -1\}$

(C) $\{(a, b, c) : a + b + c = 0\}$

(D) $\{(a, b, c) : a + b + c = 1\}$

(E) $\mathbb{R}^3$

---

24. Which of the following sets of vectors is a spanning set for the null space of the real matrix $\begin{pmatrix} 1 & 3 & -2 \\ 2 & 1 & -1 \\ -4 & -2 & 2 \end{pmatrix}$?

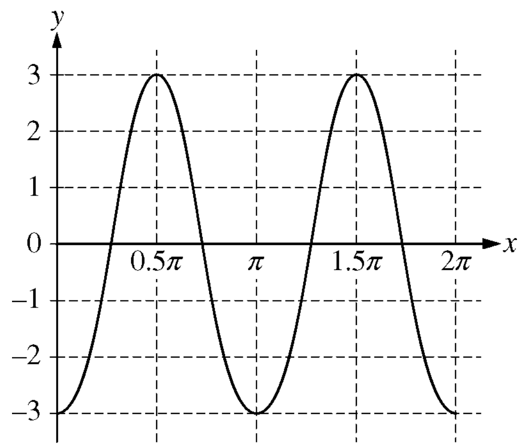
(A) $\left\{\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}\right\}$

(B) $\left\{\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}\right\}$

(C) $\left\{\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}\right\}$

(D) $\left\{\begin{pmatrix} 2 \\ -5 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}\right\}$

(E) $\left\{\begin{pmatrix} 3 \\ -5 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 10 \\ 0 \end{pmatrix}\right\}$



25. The graph of the function $f(x) = A\cos(\omega x - \phi)$ is shown in the xy-plane above, where A, ω, and ϕ are nonnegative constants. Which of the following could be the value of ϕ?

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π (E) $\frac{3\pi}{2}$

---

26. If f is the function defined by $f(x) = \max\left\{\sqrt{4 - x^2},\ x + 2\right\}$ for $-2 \le x \le 2$, what is the value of $\int_{-2}^{2} f(x)\,dx$?

(A) $2 + \pi$ (B) $4 + \pi$ (C) $6 + \pi$ (D) $2 + 2\pi$ (E) $6 + 2\pi$

---

27. Let P, Q, and R be logical propositions. Consider the statement "If P is true, then Q is true and R is true." Which of the following is the negation of the statement?

(A) P is true, and Q is false or R is false.

(B) P is false or Q is true or R is true.

(C) P is false or Q is false or R is false.

(D) If Q is true and R is true, then P is true.

(E) If P is false, then Q is false or R is false.

---

28. The value of the improper integral $\int_{-\infty}^{\infty} e^{-x^2}\,dx$ is $\sqrt{\pi}$. What is the value of $\int_{0}^{\infty} x^2 e^{-x^2}\,dx$?

(A) $\frac{\sqrt{\pi}}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) $\sqrt{\pi}$ (E) $\frac{\pi}{2}$

---

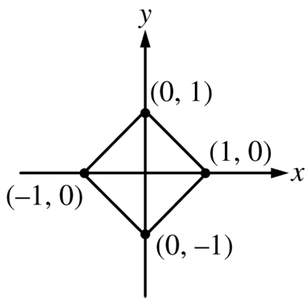
29. For all real values of x, let f be the function defined by $f(x) = \int_0^x \left(\cos^{23} y\right)\left(2 + \sin^{23} y\right) dy$. On which of the following intervals is f increasing?

(A) $(-\pi, 0)$ (B) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (C) $(0, \pi)$ (D) $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ (E) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

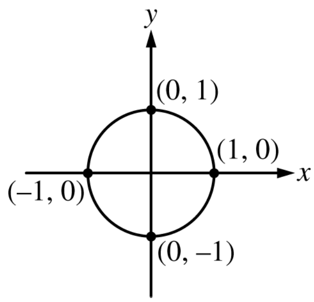
---

30. Which of the following best represents the graph of the curve $c : [0, 2\pi] \to \mathbb{R}^2$ given by $c(t) = \left(\cos^3 t, \sin^3 t\right)$ in the xy-plane?

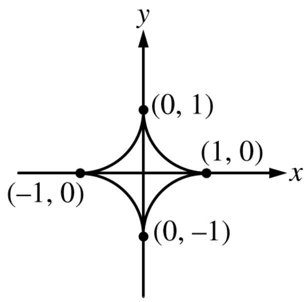
(A)



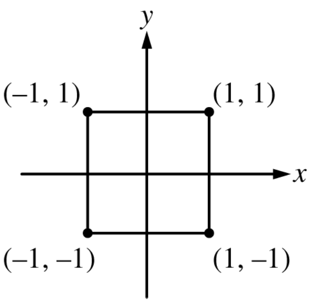
(B)



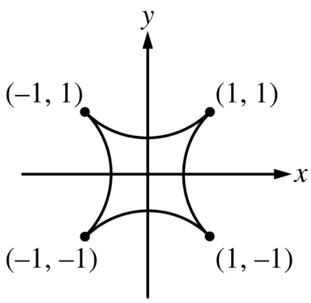
(C)



(D)



(E)



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31. Four cubes, each with faces numbered 1 to 6, are to be rolled. When each cube is rolled, each number is equally likely to appear on its top face. What is the probability that at least two of the cubes will have the same number appear on their top faces?

(A) $\frac{2}{3}$ (B) $\frac{13}{18}$ (C) $\frac{49}{54}$ (D) $\frac{23}{24}$ (E) $\frac{211}{216}$

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32. Which of the following sets of complex numbers is NOT a group under multiplication?

(A) $\{a + bi : a \text{ and } b \text{ are positive rational numbers}\}$

(B) $\left\{a + bi : a \text{ and } b \text{ are real numbers such that } a^2 + b^2 \neq 0\right\}$

(C) $\left\{a + bi : a \text{ and } b \text{ are integers such that } a^2 + b^2 = 1\right\}$

(D) $\left\{a + bi : a \text{ and } b \text{ are rational numbers such that } a^2 + b^2 = 1\right\}$

(E) $\left\{a + bi : a \text{ and } b \text{ are real numbers such that } a^2 + b^2 = 1\right\}$

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33. A spherical tank of radius 5 meters contains water that is slowly draining out. At the instant that measurements are taken, the maximum depth of the water in the tank is 2 meters, and the depth is decreasing by $\frac{1}{3}$ meter per second. What is the rate of decrease of the volume of the water, in cubic meters per second, at that instant?

(A) $\frac{16}{3}\pi$ (B) $\frac{59}{3}\pi$ (C) 22π (D) $\frac{98}{3}\pi$ (E) $\frac{176}{3}\pi$

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34. Let f be the function defined by $f(x) = 1 - \frac{1}{x}$ for all real numbers x except 0 and 1. Let $f^{\circ 2}$ denote the composition $f \circ f$, and let $f^{\circ n}$ denote the composition $f \circ f^{\circ(n-1)}$ for all integers $n \geq 3$. Then $f^{\circ 100}(x) =$

(A) $1 - \frac{1}{x}$ (B) $1 - \frac{1}{x^{100}}$ (C) x (D) $\frac{1}{1-x}$ (E) $1 + \frac{1}{x^{100}}$

---

35. Consider the sequence $\{a_n\}_{n=1}^{\infty}$, where $a_n = \sum_{k=1}^{n} \frac{n}{n^2 + k^2}$ for all $n \geq 1$. What is $\lim_{n \to \infty} a_n$?

(A) $\frac{1}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\log 2$ (E) $2 \log 2$

---

36. Let S be a bounded subset of the real line that is connected and has more than one point. Let f be a continuous real-valued function defined on S. Which of the following statements must be true?

I. S is a closed interval.

II. The set $f(S)$ has a maximum value.

III. The set $f(S)$ has a least upper bound.

(A) None (B) III only (C) I and II only (D) II and III only (E) I, II, and III

---

37. What is the maximum value of the function on $\mathbb{R}^3$ defined by $f(x, y, z) = x - 3y + 2z$ subject to the constraint $x^2 + y^2 + z^2 = 9$?

(A) $\frac{23}{\sqrt{14}}$ (B) $\frac{27}{\sqrt{14}}$ (C) $\frac{42}{\sqrt{14}}$ (D) $\frac{46}{\sqrt{14}}$ (E) $\frac{56}{\sqrt{14}}$

---

38. Let A and B be linear transformations from $\mathbb{R}^{12}$ to $\mathbb{R}^{12}$ such that the null space of A has dimension 3 and the null space of B has dimension 5. If d is the dimension of the null space of the linear transformation given by the composition $A \circ B$, which of the following indicates all of the possible values of the integer d ?

(A) $2 \le d \le 8$ (B) $3 \le d \le 5$ (C) $3 \le d \le 8$ (D) $5 \le d \le 8$ (E) $5 \le d \le 10$

$$\begin{pmatrix} 1+x & x & 1 \\ 0 & 1+x^2 & -x \\ 0 & 0 & 1-x \end{pmatrix}$$

39. In the matrix shown above, $x \in \mathbb{C}$. For how many distinct values of x is the matrix noninvertible?

(A) None (B) One (C) Two (D) Three (E) Four

---

40. Consider 10 lines in the plane such that no two of the lines are parallel and no three of the lines have a common point. The 10 lines divide the plane into how many regions?

(A) 36 (B) 45 (C) 46 (D) 55 (E) 56

---

41. Which of the following statements is true for every 3×3 matrix M with real entries?

(A) M has 3 linearly independent eigenvectors.

(B) M has at most one complex eigenvalue.

(C) M has at least one real eigenvalue.

(D) If M is invertible, then M has 3 distinct eigenvalues.

(E) If M has 2 orthogonal eigenvectors, then M has at least 2 distinct eigenvalues.

---

42. What is the area of the triangle in $\mathbb{R}^3$ with vertices $(1, 3, 2)$, $(3, 1, 2)$, and $(-2, 0, 4)$?

(A) $2\sqrt{11}$ (B) $\frac{5\sqrt{11}}{2}$ (C) $3\sqrt{11}$ (D) $\frac{7\sqrt{11}}{2}$ (E) $4\sqrt{11}$

---

43. The relation R is defined on $\mathbb{R}$ as follows. For $x, y \in \mathbb{R}$,

$$xRy \text{ if } (x - y)(xy + 2) = 0.$$

Which of the following statements are true?

I. xRx for all $x \in \mathbb{R}$.

II. If $x, y \in \mathbb{R}$ and xRy, then yRx.

III. If $x, y, z \in \mathbb{R}$ and xRy and yRz, then xRz.

(A) I only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III

---

44. Two cell phone towers, A and B, are in a flat region 10 miles apart. If a new tower S is to be located in the region such that the distance between S and A is 2 miles greater than the distance between S and B, then the locus of possible points for S is best described by

(A) a branch of a hyperbola

(B) a circle

(C) an ellipse (noncircular)

(D) a line

(E) a parabola

---

45. Let $u(x, y)$ and $v(x, y)$ be real-valued differentiable functions that are implicitly defined by the equations $x = f(u, v)$ and $y = g(u, v)$, where f and g are real-valued differentiable functions. Which of the following is an expression for $\frac{\partial u}{\partial x}$?

(A) $\frac{\partial f}{\partial u}$

(B) $\frac{\partial g}{\partial v}$

(C) $\frac{1}{\frac{\partial f}{\partial u}}$ if $\frac{\partial f}{\partial u} \neq 0$

(D) $\frac{\frac{\partial f}{\partial u}}{\frac{\partial f}{\partial u}\frac{\partial g}{\partial v} - \frac{\partial f}{\partial v}\frac{\partial g}{\partial u}}$ if $\frac{\partial f}{\partial u}\frac{\partial g}{\partial v} - \frac{\partial f}{\partial v}\frac{\partial g}{\partial u} \neq 0$

(E) $\frac{\frac{\partial g}{\partial v}}{\frac{\partial f}{\partial u}\frac{\partial g}{\partial v} - \frac{\partial f}{\partial v}\frac{\partial g}{\partial u}}$ if $\frac{\partial f}{\partial u}\frac{\partial g}{\partial v} - \frac{\partial f}{\partial v}\frac{\partial g}{\partial u} \neq 0$

---

46. Which of the following represents the general real solution $y(t)$ of the differential equation $y'' + 2y' + 3y = t$, where C_1 and C_2 denote arbitrary real constants?

(A) $C_1 e^{-t} + C_2 e^{-2t} + t$

(B) $C_1 \cos(\sqrt{2}t) + C_2 \sin(\sqrt{2}t) + t$

(C) $C_1 \cos(\sqrt{2}t) + C_2 \sin(\sqrt{2}t) + \frac{1}{3}t - \frac{2}{9}$

(D) $C_1 e^{-t} \cos(\sqrt{2}t) + C_2 e^{-t} \sin(\sqrt{2}t) + t$

(E) $C_1 e^{-t} \cos(\sqrt{2}t) + C_2 e^{-t} \sin(\sqrt{2}t) + \frac{1}{3}t - \frac{2}{9}$

---

47. What is the value of the line integral $\int_C (5x + y^3)\,dx + (3xy^2 + 8y)\,dy$, where C is a straight path in the xy-plane from the point $(2, 0)$ to the point $(0, 3)$?

(A) 0 (B) 10 (C) 26 (D) 41 (E) 46

---

48. Let f be a real-valued function defined for all $(x, y) \in \mathbb{R}^2$, and consider the graph of $z = f(x, y)$ in xyz-space. Which of the following statements are true?

I. If all the level curves of f are parallel lines, then the graph is a plane.

II. If $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist for all $(x, y) \in \mathbb{R}^2$ and both are constant, then the graph is a plane.

III. If $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ exist for all $(x, y) \in \mathbb{R}^2$ and both are identically zero, then the graph is a plane.

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

---

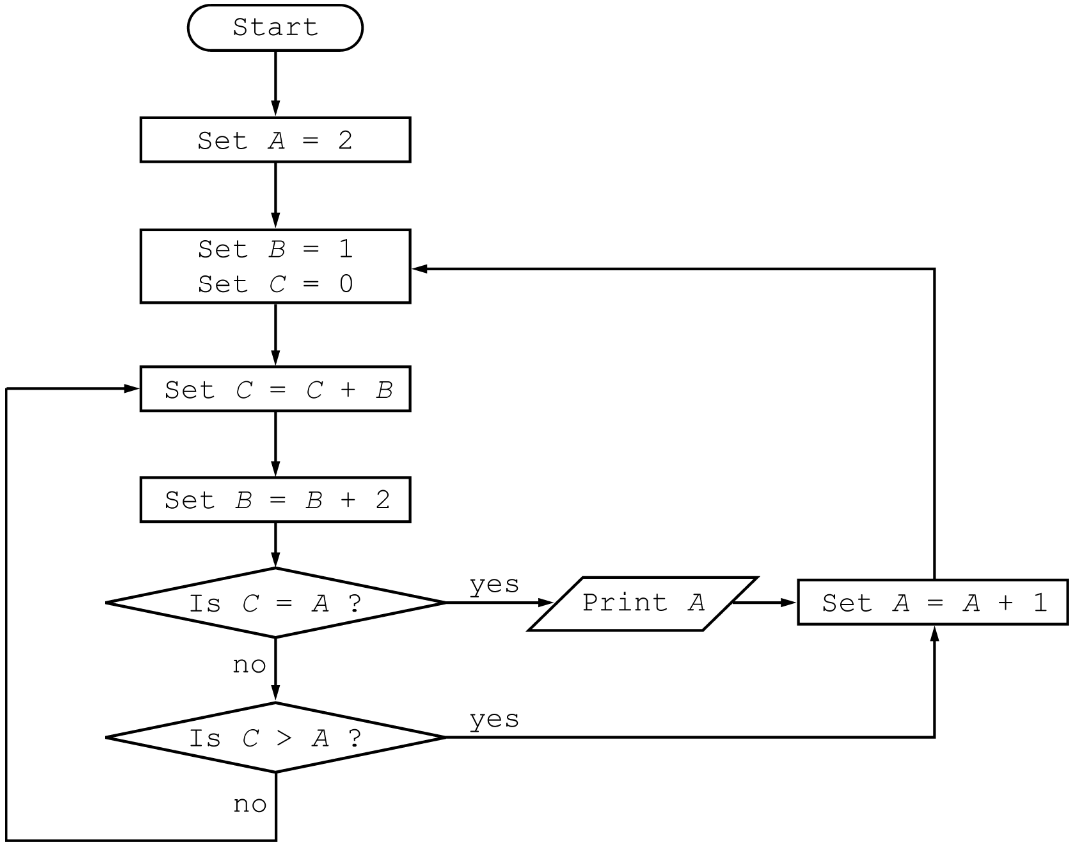
49. In the complex plane, let C be the circle $\{z = 1 + e^{i\theta} : 0 \leq \theta \leq 2\pi\}$, oriented counterclockwise. What is the value of $\frac{1}{2\pi i}\int_C \left(\frac{\sin z}{z-1}\right)^2 dz$?

(A) 0 (B) $\cos 1$ (C) $\sin 1$ (D) $\sin^2 1$ (E) $\sin 2$

---

50. $\int_0^1 \int_{2x}^2 e^{y^2}\, dy\, dx =$

(A) $\frac{1}{8}(e-1)$ (B) $e-1$ (C) $\frac{1}{2}(e^2-1)$ (D) $2(e^2-1)$ (E) $\frac{1}{4}(e^4-1)$



51. The flowchart above prints out a sequence of integers. Which of the following is a term in the sequence?

(A) 32 (B) 59 (C) 81 (D) 360 (E) 1,000

---

52. Let G be the group of permutations of 4 objects. What is the total number of conjugacy classes of elements of G ?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

53. Let $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$ be vectors in $\mathbb{R}^2$, and consider the following six dot products.

$$\mathbf{v}_1 \cdot \mathbf{v}_2 \qquad \mathbf{v}_1 \cdot \mathbf{v}_3 \qquad \mathbf{v}_1 \cdot \mathbf{v}_4 \qquad \mathbf{v}_2 \cdot \mathbf{v}_3 \qquad \mathbf{v}_2 \cdot \mathbf{v}_4 \qquad \mathbf{v}_3 \cdot \mathbf{v}_4$$

Which of the following statements about the six dot products could be true?

I. All six are negative.

II. Two are negative, and four are equal to 0.

III. Two are positive, and four are equal to 0.

(A) II only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III

---

54. Let $y(t)$ be the solution to the differential equation $y' + 2ty = e^{-t^2} \sin t$ such that $y(0) = 0$. Then $y(\pi) =$

(A) $2e^{\pi^2}$ (B) e^{π^2} (C) 0 (D) $e^{-\pi^2}$ (E) $2e^{-\pi^2}$

---

55. Let X be a continuous random variable. The standard deviation of the sampling distribution of the sample mean for random samples of 30 observations of X is equal to 8. What is the standard deviation of the sampling distribution of the sample mean for random samples of 120 observations of X ?

(A) 2 (B) 4 (C) 8 (D) 12 (E) 32

---

56. Let R be a ring with identity such that $a^2 = a$ for all $a \in R$. Which of the following statements must be true?

I. $a + a = 0$ for all $a \in R$.

II. If $b \in R$, then $b^n = 0$ for some positive integer n.

III. $ab = ba$ for all $a, b \in R$.

(A) None (B) I only (C) II only (D) I and II (E) I and III

---

57. For each integer $n \geq 1$, define the function I_n by $I_n(x) = \int_1^x (\log t)^n \, dt$ for all $x > 1$. Which of the following must be true for each $n \geq 3$ and for all $x > 1$?

(A) $I_n(x) = I_{n-1}(x) + I_{n-2}(x)$

(B) $I_n(x) = x(\log x)^n - nI_{n-1}(x)$

(C) $I_n(x) = x(\log x)^n + nI_{n-1}(x)$

(D) $I_n(x) = \frac{1}{x}(\log x)^n - nI_{n-1}(x)$

(E) $I_n(x) = \frac{1}{x}(\log x)^n + nI_{n-1}(x)$

---

58. A total of 25 identical rental trucks are to be distributed among 5 different cities. Each city can receive any number of trucks from 0 to 25, as long as the total number of trucks received is 25. For which of the following subsets of the set of all possible distributions is the number of distributions equal to $\binom{29}{4} - \binom{24}{4}$?

(A) The distributions for which at least 1 city receives 0 trucks

(B) The distributions for which each city receives at least 1 truck

(C) The distributions for which each city receives at least 4 trucks

(D) The distributions for which 4 or 5 cities receive the same number of trucks

(E) The distributions for which 4 cities receive at least 1 truck each and the other city receives 0 trucks

---

59. Let $\mathbb{Z}_{30}$ be the ring of integers modulo 30, and let U_{30} be the group of all invertible elements in $\mathbb{Z}_{30}$ under multiplication. Let ϕ be a group homomorphism from U_{30} to U_{30} with $\ker(\phi) = \{1, 11\}$. If $\phi(7) = 7$, which of the following elements does ϕ also map to 7 ?

(A) 11 (B) 13 (C) 17 (D) 19 (E) 29

---

60. A matrix M can be factored as $M = LU$, where

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{pmatrix} \text{ and } U = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 2 \end{pmatrix}.$$

If $\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{x}$ is the solution of the system $M\mathbf{x} = \mathbf{b}$, what is the first coordinate of the vector $\mathbf{x}$?

(A) −2 (B) −1 (C) 0 (D) 1 (E) 2

---

61. A primitive 10th root of unity is defined as a complex number z such that $z^{10} = 1$ but $z^k \neq 1$ for each integer k, where $1 \leq k \leq 9$. If S is the sum and P is the product of all the primitive 10th roots of unity, then

(A) $S = -1$ and $P = 1$

(B) $S = 0$ and $P = -1$

(C) $S = 0$ and $P = 1$

(D) $S = 1$ and $P = -1$

(E) $S = 1$ and $P = 1$

---

62. For each of the following metrics d, consider the metric space $(\mathbb{R}, d)$. For which of the metrics is $(\mathbb{R}, d)$ NOT a complete metric space?

(A) $d(x, y) = |x - y|$

(B) $d(x, y) = |\arctan x - \arctan y|$

(C) $d(x, y) = \left|x^3 - y^3\right|$

(D) $d(x, y) = \left|\sqrt[3]{x} - \sqrt[3]{y}\right|$

(E) $d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$

---

63. $\int_0^{\pi} \frac{\sin(100x)}{\sin x}\, dx =$

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{\pi}{2}$ (D) 50 (E) 50π

---

64. Suppose that f and g are continuously differentiable real-valued functions on $\mathbb{R}$ such that $f(0) = g(0) = 0$ and $g'(0) \neq 0$. Which of the following statements must be true?

I. The function $\frac{f}{g}$ can be extended to a continuous function in a neighborhood of 0.

II. The function $\frac{f^2 - f}{2g - g^3}$ can be extended to a continuous function in a neighborhood of 0.

III. The function $\frac{f}{g}$ can be extended to a differentiable function in a neighborhood of 0.

(A) None (B) I only (C) I and II only (D) I and III only (E) I, II, and III

---

65. In a certain population of men, 5 percent of the men have coronary artery disease. A diagnostic test is used to indicate whether or not a man has the disease, but the test is not always correct. For men who have the disease, the test is correct 24 percent of the time, while for men who do not have the disease, the test is correct 98 percent of the time. If a man is randomly chosen from the population and the test indicates that he has the disease, then the probability that the test is correct is closest to which of the following?

(A) 1% (B) 12% (C) 24% (D) 29% (E) 39%

---

66. Let $X = \{n \in \mathbb{Z} : n \geq 2\}$ be the topological space with its topology generated by the sets of the form

$$U_k = \{n \in X : n \text{ divides } k\} \text{ for } k \geq 2,$$

so that the open sets in X are the empty set and arbitrary unions of the sets U_k. For any $n \in X$, the closure of $\{n\}$ in X is

(A) $\{n\}$

(B) $\{sn : s \text{ is a positive integer}\}$

(C) $\{t \in X : t \text{ divides } n\}$

(D) $\{p : p \text{ is a prime divisor of } n\}$

(E) $\left\{\prod_{i=1}^{m} p_i : \{p_1, p_2, p_3, \ldots, p_m\} \text{ is any set of } m \text{ distinct prime divisors of } n\right\}$

Worksheet for the GRE Mathematics Practice Test, Form GR3768
Answer Key and Percentages* of Test Takers Answering Each Question Correctly

QUESTION Number	Answer	P+	CORRECT RESPONSE
1	A	87	
2	E	69	
3	D	91	
4	A	89	
5	C	73	
6	D	76	
7	B	71	
8	B	79	
9	C	76	
10	B	62	
11	E	81	
12	C	76	
13	C	62	
14	C	76	
15	A	74	
16	D	60	
17	B	83	
18	B	94	
19	C	69	
20	D	91	
21	C	72	
22	D	50	
23	E	78	
24	B	69	
25	D	57	
26	C	77	
27	A	56	
28	A	57	
29	B	70	
30	C	91	
31	B	71	
32	A	59	
33	A	48	
34	A	63	
35	B	31	

QUESTION Number	Answer	P+	CORRECT RESPONSE
36	A	37	
37	C	65	
38	D	59	
39	E	66	
40	E	50	
41	C	62	
42	A	51	
43	E	57	
44	A	61	
45	E	20	
46	E	67	
47	C	59	
48	B	27	
49	E	30	
50	E	56	
51	C	61	
52	E	20	
53	D	60	
54	E	46	
55	B	38	
56	E	46	
57	B	47	
58	A	21	
59	C	53	
60	B	30	
61	E	13	
62	B	42	
63	A	39	
64	C	35	
65	E	33	
66	B	28	

Total Correct:________________

Total Scaled: ________________

* The numbers in the P+ column indicate the percentages of test takers in the United States who answer each question correctly.

Score Conversions for the GRE Mathematics Practice Test, Form GR3768

TOTAL SCORE			
Total Correct	**Scaled Score**	**Total Correct**	**Scaled Score**
65-66	970	33	580
64	960	32	570
		31	560
63	920	30	550
62	900		
		29	540
61	890	28	530
60	880	27	520
59	870	26	510
57-58	860	25	500
56	850		
		24	490
55	840	23	480
53-54	830	22	470
52	820	21	460
51	810	20	450
50	800		
		19	440
49	790	18	430
48	770	17	420
47	760	16	410
46	750	15	400
45	740	14	390
44	720	13	370
43	710	12	360
42	700	11	350
41	680	10	330
40	670	9	320
39	660	8	300
38	650		
		7	280
37	630	6	260
36	620		
35	610	5	230
34	600	0-4	200

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